Robust Global Optimization of Electromagnetic Devices with Uncertain Design Parameters: Comparison of Worst-Case Optimization and Gradient Index Method

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Abstract —In this paper, uncertain design parameters are taken into account in the design optimization of electromagnetic devices. Two kinds of robust optimization methods, Worst-Case optimization and Gradient Index methods are reviewed. The performance and robustness are compared through a numerical experiment with the TEAM Problem 22.

I. INTRODUCTION

When an electromagnetic device is optimally designed based on numerical method such as finite element method (FEM), it often fails experimentally to give a good performance although it has an excellent performance numerically [1]. This is thought mainly due to the uncertainties in the design parameters such as manufacturing tolerance and deviation of the material constant from its nominal value.

In order to take account of these uncertainties in the design process, and increase the reliability of the optimal design, recent works in various literatures have been devoted to robust optimization [1]-[4]. Among them, Worst-Case optimization and Gradient-Index method look promising from the viewpoint of the robustness of the solution and computational efficiency.

In this paper, the Worst-Case optimization and Gradient Index method are reviewed, and their robustness and computational efficiency are compared through numerical experiments with analytic function and TEAM Problem 22.

II. ROBUST OPTIMIZATION ALGORITHMS

In an optimal design of electromagnetic devices with the aim of minimizing its objective function, we may encounter three kinds of optimal solutions as shown in Fig. 1. Each optimal solution has the following characteristics:

- Design (A): This is the global optimal design in classical (non-robust) optimization. Due to its big gradient, however, it may move up to design (A′) which has very big objective function value when the parameter has uncertainty.

- Design (B): Although this design has bigger objective function value than the design (A) and (C), its objective function value will not be worse than the design (B′) even in the worst case. In the robust optimization, this will be considered as an optimal design.

- Design (C): Although it has very small objective function value and gradient, a small perturbation in design parameter may lead to the design (C′) which has unacceptable objective function value.

In the robust optimization with uncertain design parameters, the optimization target is finding the design (B) which gives relatively good performance even in the worst case.

A. Classical Optimization

A classical (or non-robust) optimization problem with the aim of minimizing an objective function is expressed as follows:

$$\begin{aligned} &\textit{Minimize} \quad f(\mathbf{x}) \\ &\textit{subject to} \quad \mathbf{x}_L \le \mathbf{x} \le \mathbf{x}_U \end{aligned} \tag{1}$$

where $\mathbf{x}\in R^N$ is the design variable vector with N number of design parameters, and $\mathbf{x}_L$ and $\mathbf{x}_U$ are the lower and upper bounds of the design variables, respectively.

This formulation does not take into account any uncertainties in the optimization, and thus small perturbation in design parameter may increase the objective function value up to an unacceptable level.

B. Worst-Case Optimization (WCO)

This algorithm formulates the optimization problem (1) as follows:

$$\begin{aligned} &\textit{Minimize} \quad f(\mathbf{x}) \equiv \max_{\xi\in U(\mathbf{x})} f(\xi) \\ &\textit{subject to} \quad \mathbf{x}_L \le \mathbf{x} \le \mathbf{x}_U \end{aligned} \tag{2}$$

where uncertainty set, $U(\mathbf{x})$, is defined, with the assumption that the uncertainties of the design parameters are independent and follow Gaussian distribution, as follows:

$$U(\mathbf{x}) = \left\{ \xi \in R^N \,\middle|\, \mathbf{x} - k\boldsymbol{\sigma} \le \xi \le \mathbf{x} + k\boldsymbol{\sigma} \right\} \tag{3}$$

where $\mathbf{x}\in R^N$, $\boldsymbol{\sigma}\in R^N$ are nominal values and standard deviations of design parameters, respectively, and constant k will be determined according to probability. In the following paper, all design variables are considered as uncertain variables. In case of deterministic variable, the corresponding σ is set to be zero.

The WCO algorithm is burdened with a lot of computing time because it requires another optimization in uncertainty set for every trial in optimization process to calculate the worst-case objective function value. In order to reduce

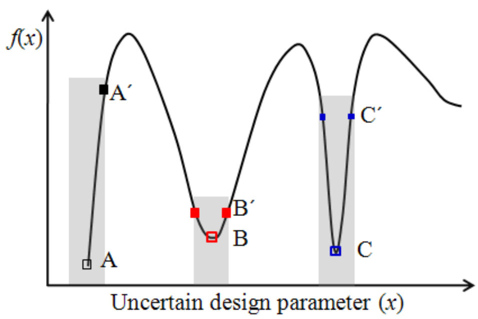

Fig. 1. Objective function to be minimized and its optimal solutions with uncertain design parameter (the gray region stands for uncertain ranges)

the computing time related with this, therefore, worst vertex prediction algorithm suggested in [4] is adopted.

The solution of (2) is found, in this paper, by using particle swarm optimization (PSO).

C. Gradient Index Method (GIM)

In *GIM*, the original problem (1) is transformed into a multi-objective optimization problem as follows:

$$\begin{aligned} &\textit{Minimize} \quad f(\mathbf{x}) \\ &\textit{Minimize} \quad GI(\mathbf{x}) = \max\left|\partial f(\mathbf{x})/\partial x_i\right|, i = 1,\cdots,N \\ &\textit{subject to} \quad \mathbf{x}_L \le \mathbf{x} \le \mathbf{x}_U \end{aligned} \quad (4)$$

where gradient index, $GI(\mathbf{x})$, is defined as the maximum component of the gradient of the objective function with respect to design parameters. The gradient vector in (4) is, in general, computed by using design sensitivity analysis based on FEM as follows [5]:

$$\frac{df}{d\mathbf{x}^T} = \left.\frac{\partial f}{\partial \mathbf{x}^T}\right|_{\mathbf{A}=C} - [\lambda]^T \left(\left.\frac{\partial \mathbf{R}}{\partial \mathbf{x}^T}\right|_{\nu=C} - \frac{\partial \mathbf{R}}{\partial \nu}\frac{\partial \nu}{\partial B^2}\frac{\partial B^2}{\partial \mathbf{x}^T} \right) \quad (5)$$

$$[K + \bar{K}]^T [\lambda] = \frac{\partial f}{\partial \mathbf{A}} \quad (6)$$

where $\mathbf{R}$ is the residual vector of Galerkin approximation in FEM, ν is the non-linear magnetic reluctivity and other symbols have the usual meanings in FEM.

The solution of (4) is found by searching Pareto-optimal solutions utilizing multi-objective particle swarm optimization (MOPSO).

III. NUMERICAL EXAMPLE AND DISCUSSIONS

TEAM 22, shown in Fig. 2, is taken as an example. In the optimization D_2, H_2 and J_1, J_2 are taken as deterministic and uncertain design parameters, respectively. The nominal values of J_1, J_2 are set to 16.78MA/m² and -15.51MA/m², respectively, and their standard deviations are assumed to be 0.179MA/m². The ranges and values for other parameters are listed in Table I.

The objective function to be minimized is defined as:

$$\textit{Minimize} \quad f(\mathbf{x}) = \frac{1}{21}\sum_{i=1}^{21}\frac{B_i(\mathbf{x})^2}{B_n^2} + \frac{|E(\mathbf{x}) - E_r|}{E_r} \quad (7)$$

where B_i is the stray field at i-th sampling point, and E_r and B_n are set to 180MJ, and 1mT, respectively. In the WCO, the uncertainty set is defined with the probability of 0.9545, i.e., k in (3) is set to 2. The gradient of the objective function is calculated as follows:

$$\nabla f(\mathbf{x}) = \frac{21}{B_n^2}\sum_{i=1}^{21} 2B_i \frac{dB_i}{d\mathbf{x}} + \frac{1}{E_r}\frac{dE}{d\mathbf{x}} Sgn\left(E - E_r\right) \quad (8)$$

where the derivatives are computed using (5) and (6).

Fig. 3 and Fig. 4 show the distribution of the Pareto-optimal designs obtained from the GIM in objective function space, design parameter space, respectively, together with the optimal design from the WCO. It can be seen that the optimal design of WCO is very near to one of the Pareto optimal design of GIM which has the objective function value and gradient index of 0.5867 and 0.0547, respectively. Therefore, results show that both WCO and GIM can improve the robustness of optimal design.

IV. REFERENCES

[1] H. T. Wang, Z. J. Liu, S. X. Chen, and J. P. Yang, "Application of Taguchi method to robust design of BLDC motor performance," *IEEE Trans. Magn*., vol. 35, no. 5, pp.3700-3702, 1999.

[2] G. L. Soares, R. L. Adriano, C. A. Maia, L. Jaulin, and J. A. Vasconcelos, "Robust multi-objective TEAM 22 problem: A case study of uncertainties in design optimization," *IEEE Trans. Magn*., vol. 45, no. 3, pp.1028-1031, 2009.

[3] N. K. Kim, et al., "Robust optimization utilizing the second-order design sensitivity information," *IEEE Trans. Magn*., vol. 46, no. 8, pp.3117-3120, 2010.

[4] G. Steiner, A. Weber, and C. Magele, "Managing uncertainties in electromagnetic design problems with robust optimization," *IEEE Trans. Magn*., vol. 40, no. 2, pp.1094-1099, 2004.

[5] J. S. Ryu, Y. Yao, C. S. Koh, S. Yoon, and D. S .Kim, "Optimal shape design of 3-D nonlinear electromagnetic devices using parameterized design sensitivity analysis," *IEEE Trans. Magn.*, vol. 41, no. 5, pp. 1792-1795, 2005.

TABLE I
VARIABLE RANGES AND VALUES USED

Variable [unit]	R_1 [m]	$H_1/2$ [m]	D_1 [m]	R_2 [m]	$H_2/2$ [m]	D_2 [m]	J_1 [MA/m²]	J_2 [MA/m²]
Min	—	—	—	—	0.1	0.1	—	—
Max	—	—	—	—	1.8	0.3	—	—
Value	1.32	1.07	0.59	1.80	—	—	16.78	-15.51

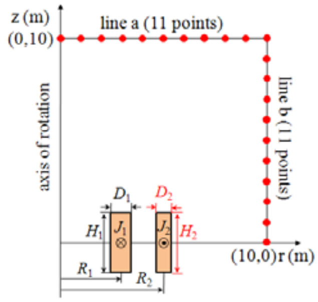

Fig. 2. Configuration of TEAM Problem 22

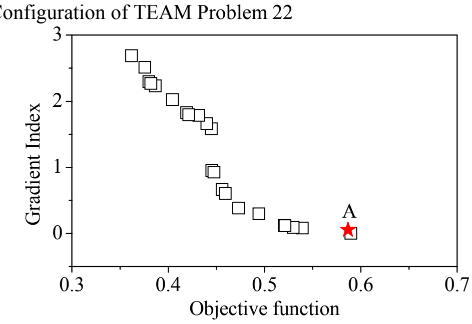

Fig.3. Pareto solutions of gradient index method

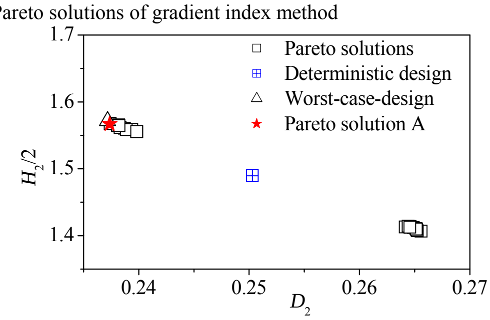

Fig.4. Distribution of Pareto design from GIM and optimum from WCO